Numerical simulations of the evolution and gravitational wave signature of a biased domain wall network in the radiation-dominated early Universe.

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Various cosmological defects arise during cosmological phase transitions accompanied by spontaneous symmetry breaking. The domain walls under consideration are formed in models with spontaneous breaking of a discrete symmetry. Such models incorporate a set of real scalar fields ϕ_i with the Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_i)^2 - V(\phi_i), \tag{1}$$

where the potential $V(\phi_i)$ possesses a discrete set of degenerate minima.In this work, we investigate the formation of a domain wall network during the radiation-dominated (RD) era within a single-scalar-field model with a spontaneously broken Z_2 symmetry. This topological defect, known as the ϕ^4 kink, arises when choosing the potential V as follows:

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - \eta^2)^2, \tag{2}$$

where λ- self-interaction constant, η- vacuum expectation value.

Introduction

For this model, an exact static solution is known:

$$\phi(z) = \eta \tanh\left((\frac{\lambda}{2})^{\frac{1}{2}} \eta z \right) \tag{3}$$

The physical thickness of such a wall is approximately equal to:

$$\delta \simeq (\sqrt{\lambda}\eta)^{-1} \tag{4}$$

Surface tension:

$$\sigma = \int T_{00} dz = \int (\phi')^2(z) dz = \frac{2\sqrt{2}}{3}\sqrt{\lambda}\eta^3 \tag{5}$$

As the Universe expands, the network of stable walls enters a scaling (self-similar) regime, reaches relativistic velocities, and, at some point, their characteristic scale becomes comparable to the horizon size. The contribution of the walls to the energy density of the Universe is given by:

$$\rho_w \sim \frac{\sigma R^2}{R^3} \sim \sigma H, \tag{6}$$

that is, it decreases more slowly than the other components and domain walls network start to dominate in the energy density of the Universe. One way to prevent this is to add a term which can be related to interaction of domain walls network with other objects, another ways are discussed by J. Garriga, A. Vilenkin and J. Zhang in the work «Black holes and the multiverse». In this work, a linear term is introduced that breaks the Z_2 symmetry:

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - \eta^2)^2 + \epsilon\eta^3\phi, \tag{7}$$

where $\epsilon > 0$ is a small parameter. The potential with an offset is shown in the figure 1.

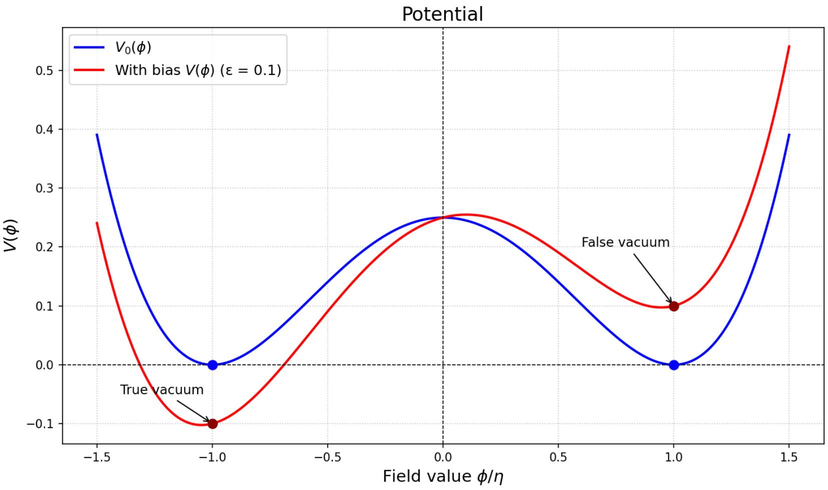


Figure 1 Potential of the model with and without bias

Producing of gravitational waves

During the evolution and subsequent decay of the domain walls, driven by the collisions and annihilation of massive spatial structures and defects-time-varying quadrupole moments of the energy-momentum tensor are produced. According to the quadrupole formula for the gravitational radiation power, $P \sim G \dddot{Q}_{ij} \dddot{Q}_{ij}$ (where Q_{ij} is the quadrupole moment of the domain walls), this process generates a strong stochastic gravitational wave background.

Producing of gravitational waves

In the work "A review of gravitational waves from cosmic domain walls" by Ken'ichi Saikawa, the following expressions were obtained:The energy density of gravitational waves generated by domain walls in the scaling regime—where $\rho_w = A\frac{\sigma}{t}$ (with A being an approximately constant parameter)—is given by:

$$\rho_{gw} \sim GA^2\sigma^2 \tag{8}$$

The dimensionless power spectrum of gravitational waves per logarithmic interval of comoving momentum is defined as:

$$\Omega_{gw}(k,\tau) = \frac{1}{\rho_c(\tau)}\frac{d\rho_{gw}}{d(\ln k)}, \tag{9}$$

which can be used to compare the simulation results with data from gravitational-wave detectors.

Numerical modeling of the evolution of the domain wall network was performed using CosmoLattice on a 1024^3 lattice. The modeling is performed in software units (denoted with a tilde):$t \to \tilde{\eta}$, $x^i \to \tilde{x}^i$, $\phi \to \tilde{\phi}$. The conversion of physical units to software units is performed as follows:

$$\tilde{\phi} \equiv \frac{\phi}{f_*}, \qquad d\tilde{\eta} \equiv a^{-1} w_* dt, \qquad d\tilde{x}^i \equiv w_* dx^i, \tag{10}$$

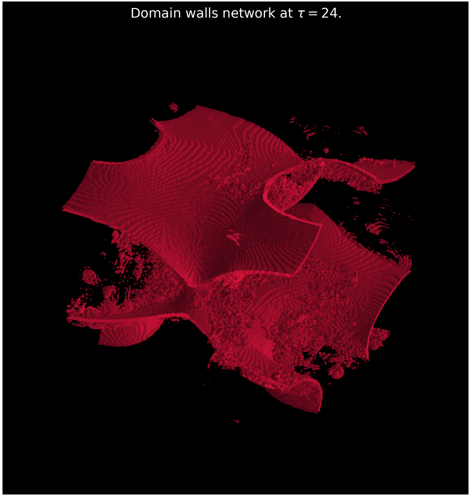
where w_*, f_* are energy dimension constants. For the ϕ^4-kinc model under consideration, the following are chosen: $f_* = \eta$, $w_* = \sqrt{\lambda}\eta$. Since the rd-stage is considered, then:

$$a(t) \sim \sqrt{t} \tag{11}$$

For convenience, it is chosen: $\tilde{\eta}_i = 1, \qquad a(\tilde{\eta}_i) = 1, \qquad H(\tilde{\eta}_i) = 1,$
where $\tilde{\eta}_i$ is the initial simulation time in software units. Substituting this into 11, we get: $a(\tilde{\eta}) = \tilde{\eta}$, $H = \frac{\dot{a}}{a} \to H(\tilde{\eta}) = \frac{1}{\tilde{\eta}^2}$.

Results

$\tau = 3$

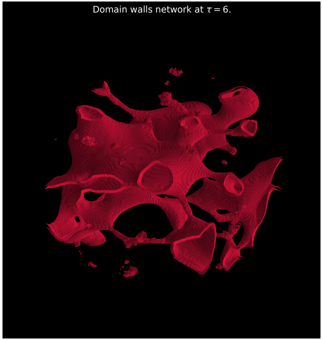


$\tau = 24$

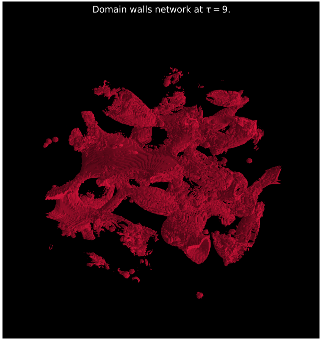
Evolution

of domain walls network with $\epsilon = 0$

Results



$\tau = 6$



$\tau = 9$

$\tau = 12$

Evolution of domain walls network with $\epsilon = 0.02$

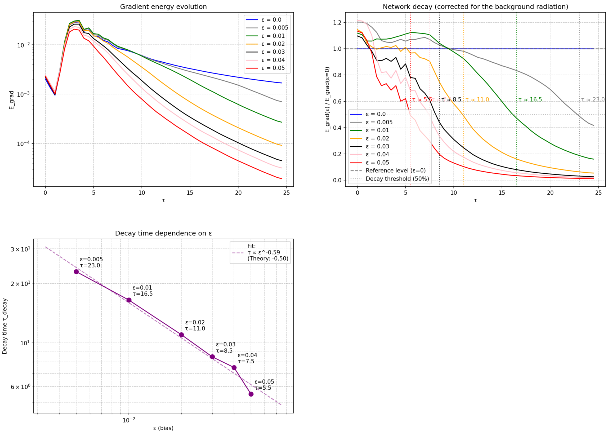


Figure 2 Gradient energy of network analysis for $\tau_f = 26$

Results

In scaling regime: $\rho_w \sim t^{-1} \implies E_G \sim t^{-1} \implies E_G \sim \tau^{-2}$ According to theory: $t_{ann} \sim \epsilon^{-1}$. According to 10, $dt = ad\tau$ and $a(\tau) = \tau \implies t \sim \tau^2 \implies \tau_{ann} \sim \epsilon^{-\frac{1}{2}}$. According to the simulation data, it was found that for the simulation with $\tau_f = 26$ $\tau_{ann} \sim \epsilon^{-0.59}$

Results

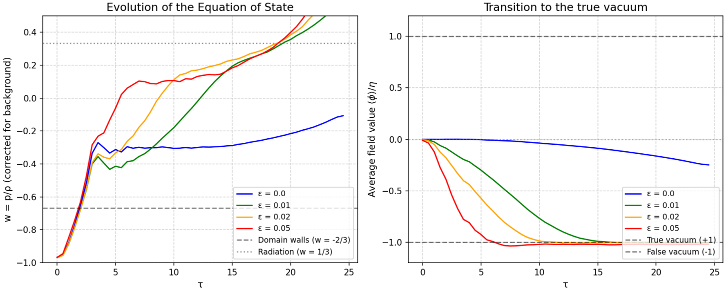


Figure 3 Thermodynamic reflection of network decay

For a pure RD stage, the equation of state is $w = \frac{p}{\rho} = \frac{1}{3}$. For a domain wall network, since in dimensionless units in CosmoLattice $\rho = E_K + E_G + V$, and $p = E_K - \frac{1}{3}E_G - V$, the kinetic energy of the walls is small, and the gradient and potential energies are approximately equal, $w = -\frac{2}{3}$ is characteristic

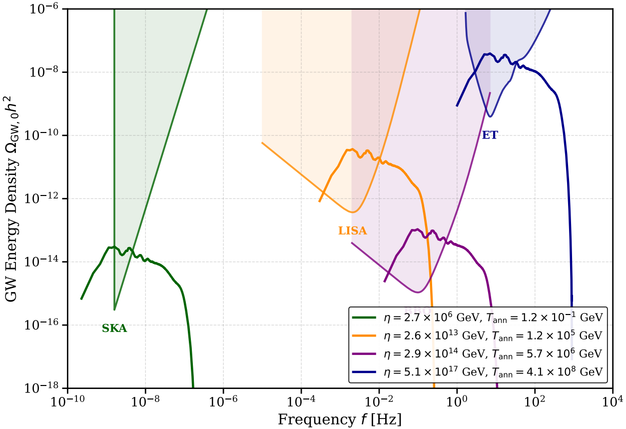


Figure 4 Stochastic gravitational wave background (SGWB) spectra from a decaying network. The spectral shapes are extracted from 3D CosmoLattice simulations and projected onto the sensitivity curves of various detectors

Results

The present-day peak frequency of the stochastic gravitational wave background (SGWB), f_0, is determined by the plasma temperature at the moment of network decay:

$$f_{peak} \approx \frac{k_{peak}}{2\pi} = \frac{x_{peak} H_a}{2\pi} = \frac{x_{peak}}{2\pi} \times 1.1 \times 10^9 Hz \left(\frac{g_*(T_a)}{10}\right)^{\frac{1}{2}} \left(\frac{g_{*s}(T_a)}{10}\right)^{-\frac{1}{3}} \left(\frac{T_{ann}}{10^{-2} GeV}\right), \tag{12}$$

where g_* and g_{*s} are the effective relativistic degrees of freedom for the energy density and the entropy density, respectively. The relationship between the present-day physical SGWB energy density ($\Omega_{\mathrm{GW},0} h^2$) and the peak amplitude on the lattice is given by:

$$\Omega_{GW,0} h^2 = \Omega_{R,0} h^2 \left(\frac{g_*(T_{ann})}{g_{*,0}}\right) \left(\frac{g_{*s,0}}{g_{*s}(T_{ann})}\right)^{\frac{4}{3}} \left[E^{(CL)}_{peak} \frac{\omega_*^4}{3 M_{Pl}^2 H_{ann}^2}\right], \tag{13}$$

where $\Omega_{R,0}$- modern radiation energy density, $\Omega_{R,0} h^2 = 4.15 \times 10^{-5}$, $E^{(CL)}_{peak}$- dimensionless peak of amplitude of GW, calculated by CosmoLattice.

Results

Comparing calculated values with sensitive curves of gravitational waves detectors physical scale of network was calculated (T_{ann}, w_*):

Detector	T_{ann} [GeV]	Scale η [GeV]	t_{ann}, sec
SKA	1.2×10^{-1}	2.7×10^{6}	5.2×10^{-5}
LISA	1.2×10^{5}	2.6×10^{13}	1.5×10^{-17}
BBO	5.7×10^{6}	2.9×10^{14}	7×10^{-21}
ET	4.1×10^{8}	5.1×10^{17}	1.3×10^{-24}

Conclusion

- The transition of the unperturbed domain wall network to the scaling regime has been confirmed.
- The efficiency of the wall annihilation mechanism upon introducing a small perturbation breaking the Z_2 symmetry has been demonstrated.
- It has been established that the dependence of the conformal decay time on the perturbation magnitude $\tau_{ann} \sim \epsilon^{-0.59}$ is in good agreement with the theoretical prediction $\tau_{ann} \sim \epsilon^{-\frac{1}{2}}$.
- The evolution of the macroscopic equation of state of the system $w = \frac{p}{\rho}$ has been tracked from the domain wall network domination regime to the radiation-dominated phase with characteristic values of w.
- It has been confirmed that the generation of gravitational waves ceases upon the decay of the wall network, forming a frozen relic background. After comparing the gravitational-wave spectrum with the detector sensitivity curves, we obtained the viable energy scales as well as the physical annihilation time of the domain wall network for which the network could account for the observed spectra.